

WRITTEN EXAM AT THE DEPARTMENT OF ECONOMICS
WINTER 2018–19

ADVANCED MICROECONOMETRICS

FINAL EXAM

January 2nd, 2019 (13:00–16:00)
(3-hour closed book exam)

Answers only in English.

This exam question consists of 6 pages in total.

NB: If you fall ill during an examination at Peter Bangs Vej, you must contact an invigilator who will show you how to register and submit a blank exam paper. Then you leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Be careful not to cheat at exams!

You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed.
- Communicate with or otherwise receive help from other people.
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text.
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts.
- Or if you otherwise violate the rules that apply to the exam.

Note: The percentage weights indicated on each problem should only be regarded as indicative. The final grade will ultimately be based on an overall assessment of the quality of the answers to the exam questions in their totality.

Problem 1 (40%)

Consider the following random utility model, for a sample of N individuals:

$$y_i = \arg \max_{j \in \{0,1\}} \{u_{ij}\}, \quad i = 1, \dots, N, \quad (1)$$

$$u_{ij} = x_i' \beta_j + \varepsilon_{ij}, \quad \text{for } j = 0, 1, \quad (2)$$

where the explanatory variables contained in the $K \times 1$ vector x_i influence each level of utility through the $K \times 1$ vector of regression coefficients β_j , for $j = 0, 1$.

The error terms are assumed to be independent and identically distributed across observations, with a joint normal distribution:

$$\begin{pmatrix} \varepsilon_{i0} \\ \varepsilon_{i1} \end{pmatrix} \stackrel{iid}{\sim} \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{bmatrix} \right). \quad (3)$$

Question 1.1: Show that the probability of choosing alternative 1, for each individual $i = 1, \dots, N$, is equal to

$$\Pr(y_i = 1 \mid x_i, \theta) = \Phi \left(\frac{x_i'(\beta_1 - \beta_0)}{\sqrt{\sigma_0^2 + \sigma_1^2 - 2\sigma_{01}}} \right), \quad (4)$$

where $\theta = (\beta_0', \beta_1', \sigma_0^2, \sigma_1^2, \sigma_{01})'$, and $\Phi(\cdot)$ denotes the cumulative distribution function (CDF) of the standard normal distribution.

Question 1.2: Using Eq. (4), derive the corresponding log-likelihood function of the model for the whole sample of N individuals.

Question 1.3: Discuss the identification of the model. In particular, explain precisely which parameter(s) can be identified, and which restrictions, if any, are required to achieve identification.

Question 1.4: Using the identification strategy discussed in Question 1.3, show that this random utility model with two alternatives can be expressed as a standard probit model. State the corresponding probit model as part of your answer.

Problem 2 (40%)

Consider the following linear regression model with two scalar regressors x_1 and x_2 , for $i = 1, \dots, N$:

$$y_i = x_{1i}\beta_1 + x_{2i}\beta_2 + \varepsilon_i, \quad \varepsilon_i \stackrel{iid}{\sim} \mathcal{N}(0, \tau^{-1}), \quad (5)$$

where the precision parameter $\tau = 1/\sigma^2$ is the inverse of the variance of the error term. The observations are collected in the vectors $y = (y_1, \dots, y_N)'$, $x_1 = (x_{11}, \dots, x_{1N})'$ and $x_2 = (x_{21}, \dots, x_{2N})'$.

The parameters of the model $\theta = (\beta_1, \beta_2, \tau)'$ are assumed to be a priori independent, such that $p(\theta) = p(\beta_1, \beta_2, \tau) = p(\beta_1)p(\beta_2)p(\tau)$. An improper prior is assumed on the regression coefficients, and a Gamma distribution on the precision parameter:

$$p(\beta_1) \propto 1, \quad p(\beta_2) \propto 1, \quad \tau \sim \mathcal{G}(a_0, b_0), \quad (6)$$

with $a_0 > 0$ and $b_0 > 0$, where the probability density function of the Gamma distribution is

$$p(\tau | a_0, b_0) = \frac{1}{\Gamma(a_0)b_0^{a_0}} \tau^{a_0-1} \exp\left\{-\frac{\tau}{b_0}\right\}, \quad (7)$$

with $\Gamma(\cdot)$ denoting the Gamma function.

Question 2.1: Without deriving any conditional distributions, outline the steps of a Gibbs sampler that can be implemented to draw the three parameters of the model iteratively (i.e., in three different steps).

Be as precise as possible in the description of the sampler.

Question 2.2: Derive the conditional distribution $p(\tau | y, x_1, x_2, \beta_1, \beta_2)$.

Explain the concept of natural conjugacy, and explain why the Gamma distribution assumed on τ is (or is not) a natural conjugate prior in this model.

Three different data sets with $N = 100$ observations and different levels of correlation between the two regressors, $\rho \equiv \text{corr}(x_1, x_2) \in \{0.5, 0.9, 0.99\}$, are generated from the model specified in Eq. (5).

The three-step Gibbs sampler outlined in Question 2.1 is run for 1,000 iterations on each of these three data sets, with the same prior specification in the three cases. The trace plots and autocorrelograms of the parameter β_1 are displayed in Fig. 2.1 for the three cases.

Question 2.3: Match each of the three cases shown in Fig. 2.1 to the three data sets (i.e., the three different values of ρ). Explain intuitively the differences observed between these three MCMC outputs.

Question 2.4: Explain precisely if and how you can use the random draws of β_1 shown in Fig. 2.1 to draw posterior inference about this parameter in each of the three cases.

If these draws cannot be used, what would you have to change in the implementation of the Gibbs sampler to be able to do posterior inference on the parameters?

[Note: You do not need any additional information about the data generating process, about the prior specification or about the configuration of the Gibbs sampler to answer the last two questions.]

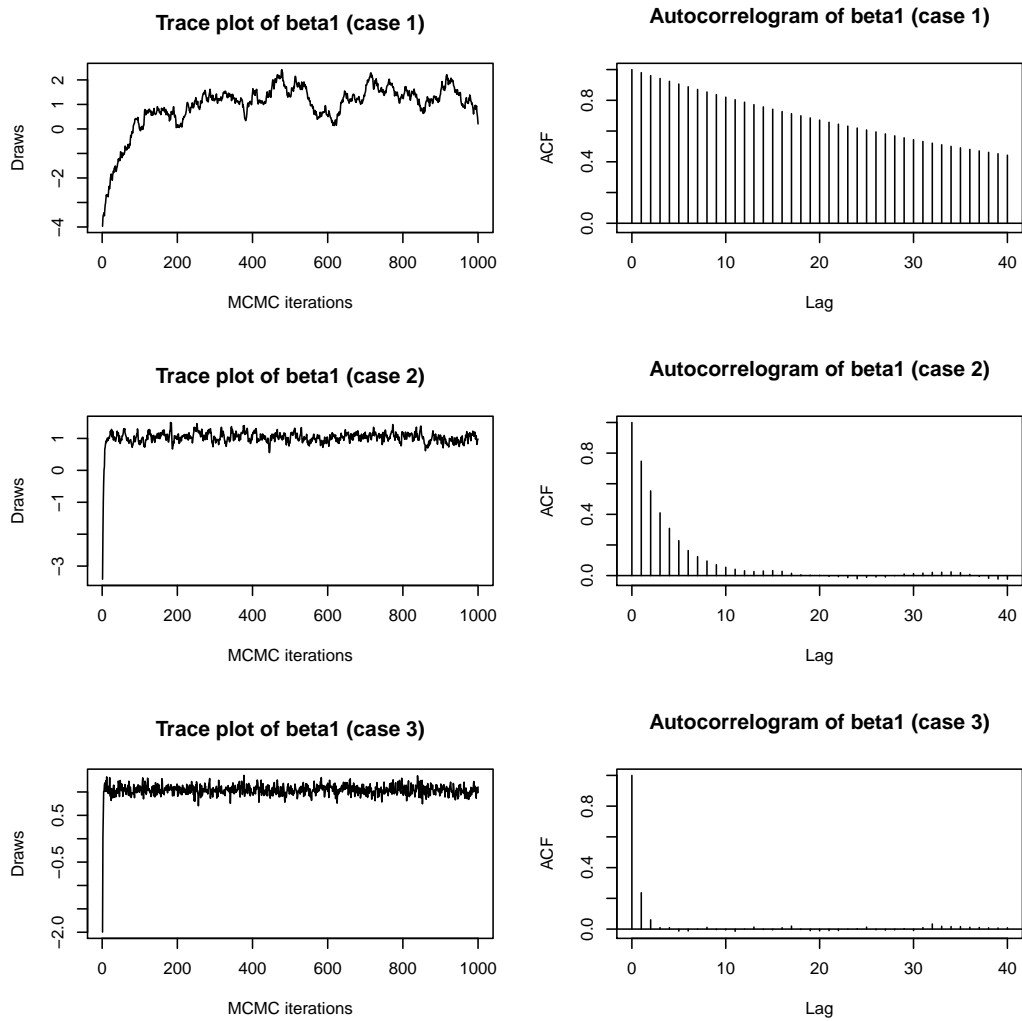


Figure 2.1: Trace plots and autocorrelograms of the parameter β_1 for the three different cases.

Problem 3 (20%)

Consider the following MATLAB function:

```
1 function [y,X,theta] = gen_data(N,J,K)
2     theta = 2*rand(K,J-1)-1;
3     X = [ones(N,1), sqrt(2).*randn(N,K-1)];
4     V = [zeros(N,1), X*theta];
5     E = gevinv(rand(N,J));
6     U = V + E;
7     [maxU,y] = max(U,[],2);
8 end
```

Question 3.1: Express in mathematical terms what this function does. You should just provide a few equations to answer this question. Be explicit about the notation.

[Note: The MATLAB function `gevinv()` computes the inverse of the CDF of the standard Gumbel distribution (type 1 extreme value distribution).]

Question 3.2: Describe briefly the econometric model that can be used to fit the corresponding data, as well as an estimation method that can be implemented to estimate the unknown parameters θ .